Generalized Gilbert Varshamov Codes

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We introduce a random coding technique for transmission over discrete memoryless channels, reminiscent of the basic construction attaining the Gilbert-Varshamov bound for codes in Hamming spaces. The code construction is based on drawing codewords recursively from a fixed type class, in such a way that a newly generated codeword must be at a certain minimum distance from all previously chosen codewords, according to some generic distance function. We derive an achievable error exponent for this construction, and prove its tightness with respect to the ensemble average. We show that the exponent recovers the Csiszár and Körner exponent as a special case, which is known to be at least as high as both the random-coding and expurgated exponents, and we establish the optimality of certain choices of the distance function. In addition, for additive distances and decoding metrics, we present an equivalent dual expression, along with a generalization to non-finite alphabets via cost-constrained random coding.